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SIMPLIFYING CONSIDERATIONS ON THE ASEISMIC DESIGN OF SHEAR STRUCTURES

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CHAPTER 6 : Earthquake Resistant Design

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SIMPLIFYING CONSIDERATIONS ON THE ASEISMIC DESIGN OF SHEAR STRUCTURES

by

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S U M M A R Y

When dealing with shear structures,where the "shear building hypothesis" is maintained the designer can choose between the following two cases for modeling the structure: (a) Lumped mass system, and (b) Continuous or distributed mass system.

In the present paper the theory for distributed mass systems of shear type is applied by an adequate computer program to a 26-story model shear structure,and the maximum dynamic displacements and story shears has been computed.

By an other computer program the maximum dynamic response of the 26-story shear structure has been computed as a lumped mass system. Several lumped mass systems have been used by concentrating the story masses either in every central story line or in every second or in every third,etc.

A very good agreement of the results between the abovedescribed models has been observed,using an envelop of response spectra for the Athenian bed rock.

I N T R O D U C T I O N

Let us have a 26 story model structure,the plan of which is presented in Fig.1. The fundamental assumption throughout this paper is the shear behaviour of the structure. When the designer has to evaluate its seismic response and specially to calculate its probable maximum story drifts and shear forces, he has basically to choose between the foolowing two models:

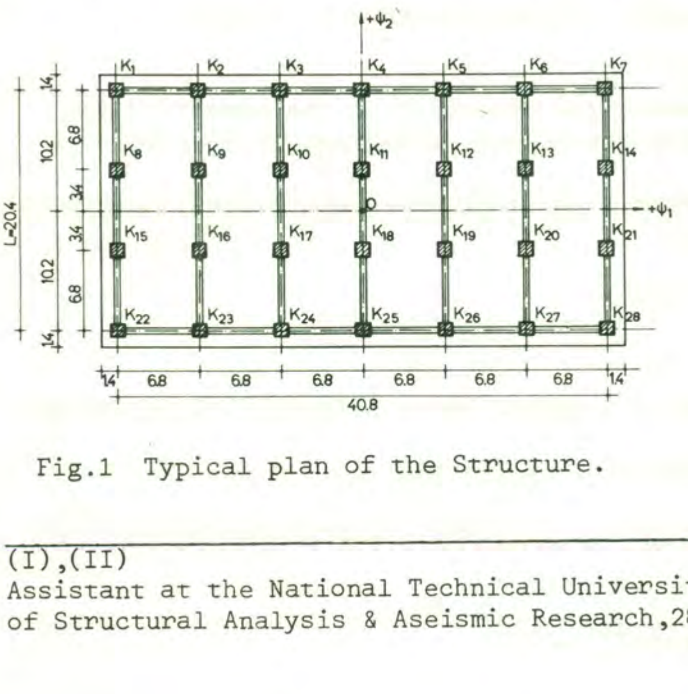

Fig.1 Typical plan of the Structure.

1) A lumped mass system with as many concentrated masses as the number of the stories of the structure, as is shown in the Fig.2 under the heading "26" and
2) A distributed mass system with varying cross section.

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Under the theory of modal superposition the designer can solve his problem,but he always needs a small computer to facilitate his calculations. But when these means are limited,the authors propose the following two simplifications:

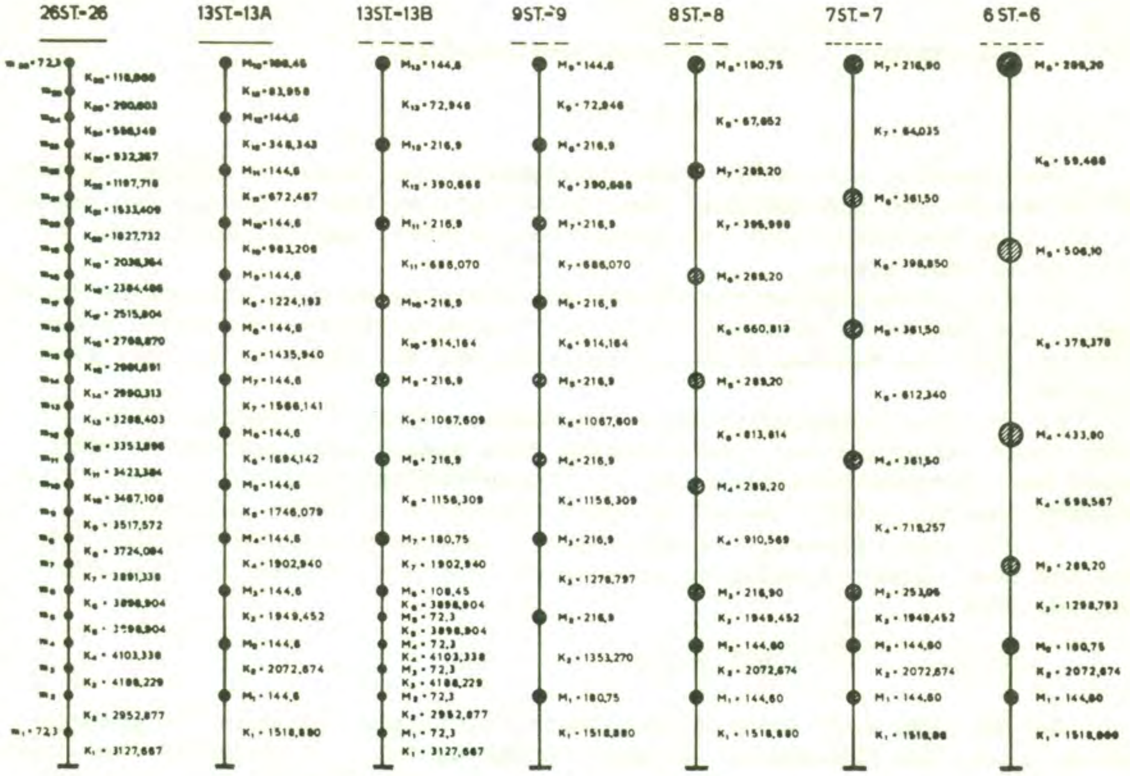


Fig.2 The original 26 concentrated mass system and 6 other simplified models.

1.1) A lumped mass system with about the one-third of the masses of the original system,reducing thus the number of degrees of freedom by a factor of about three,and

2.1) A distributed mass system with a constant cross section with adequate dimensions.

THE BACKGROUND THEORY

1) Lumped Mass Systems.

From the well known system of n second order differential equations:

$$[m]\{\ddot{v}(t)\}+[c]\{\dot{v}(t)\}+[K]\{v(t)\} = -\ddot{y}_o(t)\{m\} \qquad (1)$$

the probable maximum translation of the i story is calculated by the expression:

$$\max v_i(t) = \sqrt{\sum_{r=1}^{n} \left(\Delta_{ir}\psi_r \max\gamma_r(t)\right)^2} \qquad (2)$$

where Δ_{ir}: the amplitude of the i story at the r mode.

$\psi_r = \sum_{i=1}^{n} \Delta_{ir} m_i$: participation factor of the r mode and

$\max\gamma_r(t)$: the spectral amplitude.

For the probable maximum dynamic shear forces the expression:

$$\max Q_i(t) = \sqrt{\sum_{r=1}^{n} \left(\max Q_{ir}(t)\right)^2} \tag{3}$$

is used, where:

$$\max\{Q(t)\}_r = [\Sigma][K]\{\Delta\}_r \psi_r \max\gamma_r(t) \tag{4}$$

in which $[\Sigma]$ is the summation matrix, $[K]$ the stiffness matrix and $\{\Delta\}_r$ the r normal mode vector.

In order to reduce the total number n of degrees of freedom, the "SERAC, 1964" studies are proposed by the authors. According to these studies a new substitute system for the original one is established having n′ degrees of freedom (n′< n). The new story stiffness index $K'_{i'}$ of the i′ new level, which substitutes the i-λ, i-λ+1,…i-1 and i story of the original structure is given by the formula:

$$K'_{i'} = \sum_{j=o}^{\lambda} \frac{1}{K_{i-j}} \tag{5}$$

and the new mass $m'_{i'}$ is computed by the low of computing the support reactions of the simply supported beams i′ ÷ i′-1 and i′÷ i′+1, having loads the original masses m_i.

Six in total such a substitute systems have been considered as shown in Fig.2, under the indications 13A, 13B, 9, 8, 7 and 6 denoting thus, the total number of masses to which the original 26-story mass system has been substituted. For every case its eigenvalues ω_r and eigenvectors $\{\Delta\}_r$ have been computed. The corresponding first eight eigenperiods $T_r = 2\pi/\omega_r$, where applicable, are shown in Table I. A very good agreement is observed for the 13A, 13B, 9 and 8 mass systems, and especially for their first three periods. The forcing functions $\Delta_{ir}\psi_r$ are shown in Fig.3a,b,c for the first, second and third mode respectively.

TABLE I

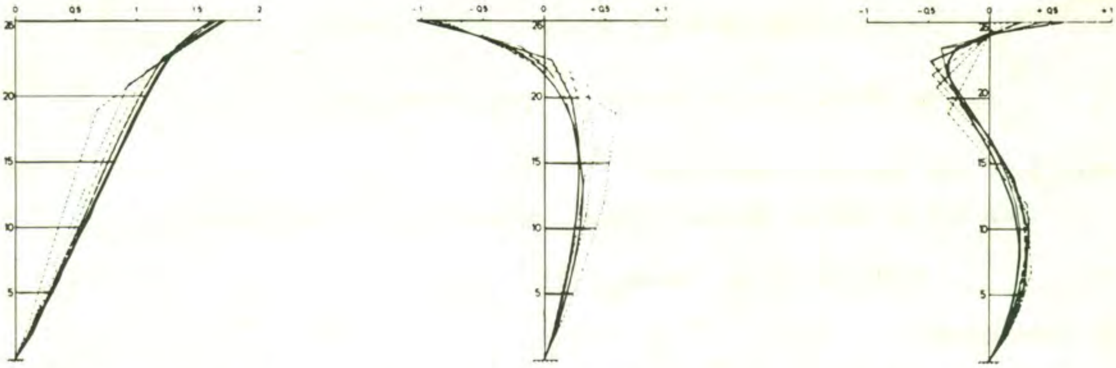
	26	13A	13B	9	8	7	6
1	1,118	1,124	1,136	1,137	1,157	1,184	1,252
2	0,511	0,545	0,590	0,591	0,633	0,665	0,700
3	0,332	0,354	0,360	0,361	0,359	0,353	0,345
4	0,240	0,248	0,243	0,243	0,239	0,234	0,210
5	[illegible]	0,188	0,181	0,181	0,179	0,179	0,124
6	0,154	0,149	0,142	0,144	0,143	0,104	0,0729
7	0,128	0,123	0,117	0,120	0,100	0,066	
8	0,110	0,105	0,101	0,103	0,066		

Table I. The first eight eigen periods T_r

In order to evaluate the response of the assumed systems a maximum ground acceleration is defined after "Galanopoulos 1971" according to the formula:

$$a_g = 0.26-0.1I+0.01I^2 \tag{6}$$

which, for an intensity I = 5, gives: $a_g = 0.01g$. After "Kokinopoulos and Carydis 1973", the response spectra of simulated bed rock motions for the region of Athens have been used, the velocity and acceleration of which are



a: First Mode b: Second Mode c: Third Mode

Fig.3 Forcing Functions $\Delta_{ir}\psi_r$

shown in Fig.4 and 5 respectively,for a damping ratio ζ = 5%. The probable maximum story displacements of the various systems is presented in Fig.6.

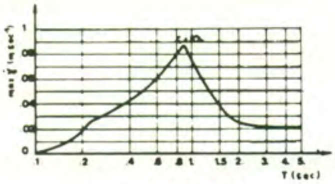


Fig.4 Velocity Spectrum

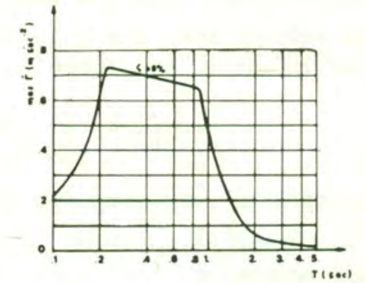


Fig.5 Acceleration spectrum

The top max displacement is equal to about 2,2cm. In fig.7 the probable maximum story accelerations of the various systems is shown. The top max acce-

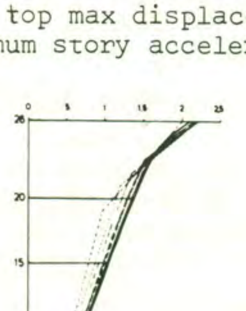


Fig.6 Max story Displacements

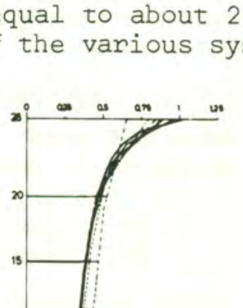


Fig.7 Max st .y Accelerations

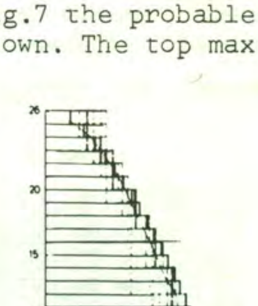


Fig.8 Story Shears

leration is about $1msec^{-2}$. The probable max story shears of every system are shown in Fig.8. A very good agreement in Figs 6,7 and 8 is observed among the 13A,13B,9 and 8 mass systems.

2) Distributed Mass Systems.

If A(x) is the cross section at the level x,being within the i story, it is easy to prove that,for shear deformations only,it must be:

$$k'A(x) = K_i H_i / G \tag{7}$$

where k′is the cross section coefficient,combining the maximum with the mean shear stress, H_i the height of the i story,K_i the story index for the lumped mass system and G = 800000tm^{-2}. Using eq.(7) for i = 1 to n, the equivalent distributed mass system,with a constant mass per unit lenght equal to m = = 72.3•26/89.6 = 20.98$tm^{-2}sec^2$ and the product **k**′ A(x) are defined.

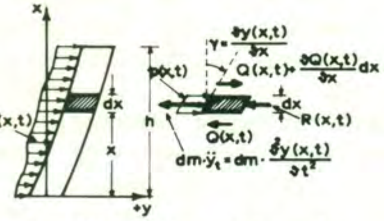


Fig.9 Model for Distributed mass Systems, Shear Type.

For the mass element dm = m(x)dx = = ρA(x)dx, in which ρ is the mass density, the Newton's low gives for a base seismic motion:

$$\rho A(x)\,\frac{\partial^2 y(x,t)}{\partial t^2} - \frac{\partial Q(x,t)}{\partial x} + R(x,t) = -\rho A(x)\ddot{y}_o(t) \tag{8}$$

in which R(x,t) is the viscous type damping force: $R(x,t) = c\,\frac{\partial y(x,t)}{\partial t}$. Using the modal superposition low:

$$y(x,t) = \sum_{r=1}^{\infty} \varphi_r(x)\xi_r(t) \tag{9}$$

eq.(8) gives finally the dynamic translations of the structure as:

$$y(x,t) = -\sum_{r=1}^{\infty} \frac{\varphi_r(x)}{\omega_r}\,\psi_r^* \int_0^t \ddot{y}_o(\tau) e^{-\zeta_r\omega_r(t-\tau)} \sin\{\omega_r(t-\tau)\}\,d\tau \tag{10}$$

where the participation factor:

$$\psi_r^* = \int_0^L \rho A(x)\varphi_r(x)dx \Big/ \int_0^L \rho A(x)\varphi_r^2(x)dx \tag{11}$$

The max shear force $Q_r(x)$ of the mode r is given by:

$$\max Q_r(x) = k'A(x)G\,\frac{d\varphi_r(x)}{dx}\,\frac{\psi_r^*}{\omega_r}\,\max\dot{\gamma}_r \tag{12}$$

and the probable maximum values are obtained by applying again the r.m.s low of eq.(3).

In order to obtain the first three periods and mode shapes the authors compiled a computer program according to the Rayleigh's principle:

$$\omega^2 = g\int_0^L \frac{\max Q^2(x,t)}{k'GA(x)}\,dx \Big/ \int_0^L q(x)y^2(x)dx \tag{13}$$

and the Rayleigh-Ritz method according to the following system of three equations for r = 1,2 and 3:

$$k'^3 \int_0^L A(x) G \frac{d\varphi(x)}{dx} \frac{\partial\left(\dot{\varphi}(x)\right)}{\partial\psi_r'} dx - \frac{\omega_r^2}{g} \int_0^L q(x)\varphi(x) \frac{\partial\varphi(x)}{\partial\psi_r'} dx = 0 \qquad (14)$$

where:

$$\varphi(x) = \psi_1'\varphi_1'(x)+\psi_2'\varphi_2'(x)+\psi_3'\varphi_3'(x) \qquad (15)$$

The first three normal modes,received thus,are depicted in Fig.10. The corresponding periods are almost identical to the values of the original 26 story given in the first column of Table I. The probable maximum shear forces are shown with continuous line in Fig.8,the difference in the top,relying in the fact that not any horizontal force at this point has been additionally assumed.

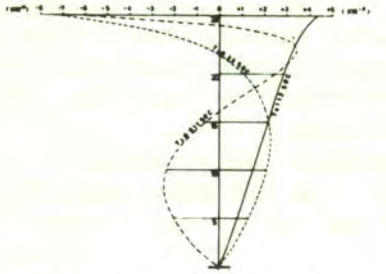


Fig.10 The first three normal modes for the distributed mass system.

It must be emphasized that assuming a constant cross section throughout the height of the structure equal to the mean $k'A(x) = 2{,}4m^2$, and a mean mass density $\rho = 8{,}74\ tm^{-4}sec^2$, the periods:

$$T_r = \frac{2\pi}{\omega_r} = \frac{4L}{(2r-1)} \sqrt{\frac{\rho}{k'G}} = \frac{1{,}3}{2r-1} \sec \qquad (16)$$

are respectively: $T_1 = 1{,}3$sec, $T_2 = 0{,}43$sec, $T_3 = 0{,}26$sec, being in good agreement with the values of the original 26 story, see Table I.

The authors assumed constant,taken graphically,mean cross sections for every five or six stories. The results concerning the periods and max-shears,are also in a good agreement with the original ones,while the calculations are extemily limited.

C O N C L U S I O N S

It is rather sufficient for engineering practices to concentrate the masses at every three or four stories.

The continuous mass system with a mean constant cross section gives, for the periods of the system,very good results. A tapered building with cross sections equal to the mean of five or six stories,gives good results for periods and probable maximum shears.

A close formula for a pyramid building after "Korchinskiy 1960" is under evaluation by the authors,in an efford of a further simplification of the problem.

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